Applications of Bayesian Statistical Analyses in Determining the Number of Demonstration Tests to Conduct and in Monitoring Reliability Growth

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The Basic Problem Addressed

- **A new system such as a new spacecraft is to be evaluated for its reliability***
- **Part of the evaluation involves determining the number of tests to perform before acceptance**
- **The evaluation also involves dynamically tracking the reliability evolution of the system with test and operation**
- **To optimize resources, the evaluations need to utilize all available information**
- **Uncertainties also need to be treated and be quantified**

***Safety is treated as part of reliability here**

Basic Concepts: Design Reliability and Demonstration Tests

- **Design reliability is the probability that a new system has no inherent failure-causing faults**
- **Demonstration tests are conducted to detect any such inherent failure-causing faults**
- **Demonstration tests can be partial tests or can be test flights**
- **A major issue: How tests are needed to demonstrate a given reliability to a given certainty?**

Factors Determining Number of Tests to Conduct

- From reliability growth principles the required number of tests depends on three major factors:
  - Initial Assurance Level
  - Fault-Detection Effectiveness
  - Corrective Action Effectiveness
- Objective: Develop an approach that incorporates these factors and quantifies the reliability after a given number of tests

Framework of the Bayesian Approach

- The reliability estimate is described by a probability distribution to account for uncertainties
- The distribution gives the mean, median, and uncertainty bounds
- An initial distribution (prior distribution) is constructed to account for initial knowledge
- The distribution is updated from the results of a test using Bayes theorem
- This updating is continued to determine the number of required tests or to track performance

Advantages of the Bayesian Approach

- The Bayesian approach can utilize both quantitative and qualitative information
- Uncertainties are comprehensively quantified
- Assessments are dynamically updated as information is gained from the tests
- The Bayesian approach is standardly used in NASA risk and reliability assessments
- Software exists that can carry out the evaluations in an efficient manner

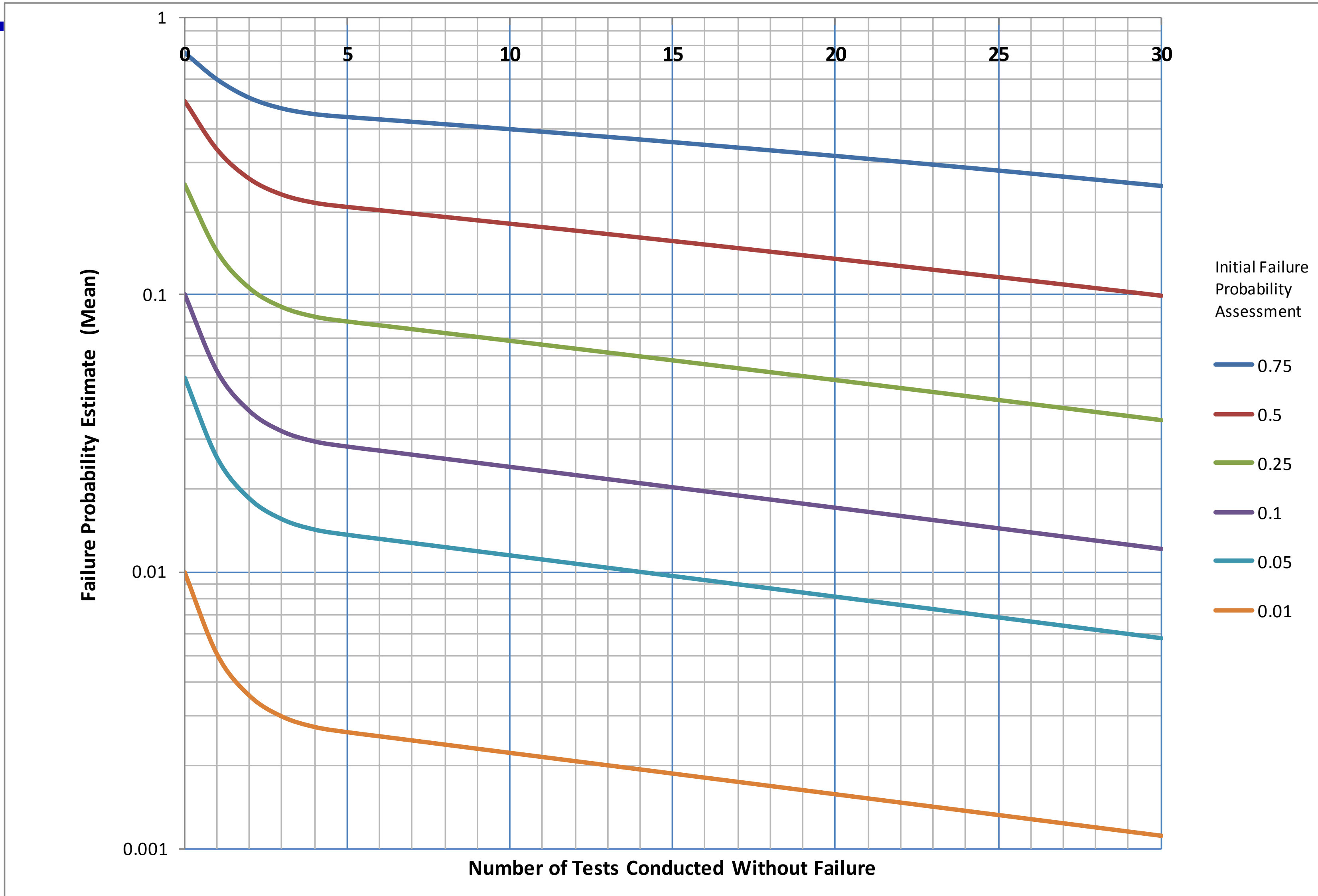
Inputs to the Bayesian Approach

- The *Prior System Reliability Estimate* is determined from the bases for the Initial Assurance Level:
  - Hazard analyses and FMEAs
  - Reliability and Risk analyses
  - Oversights and Reviews
- The *Fault Detection Probability* and the *Fault Correction Probability Estimates* are determined using test and repair information:
  - System specific data
  - Shuttle analyses and data
  - Constellation analyses

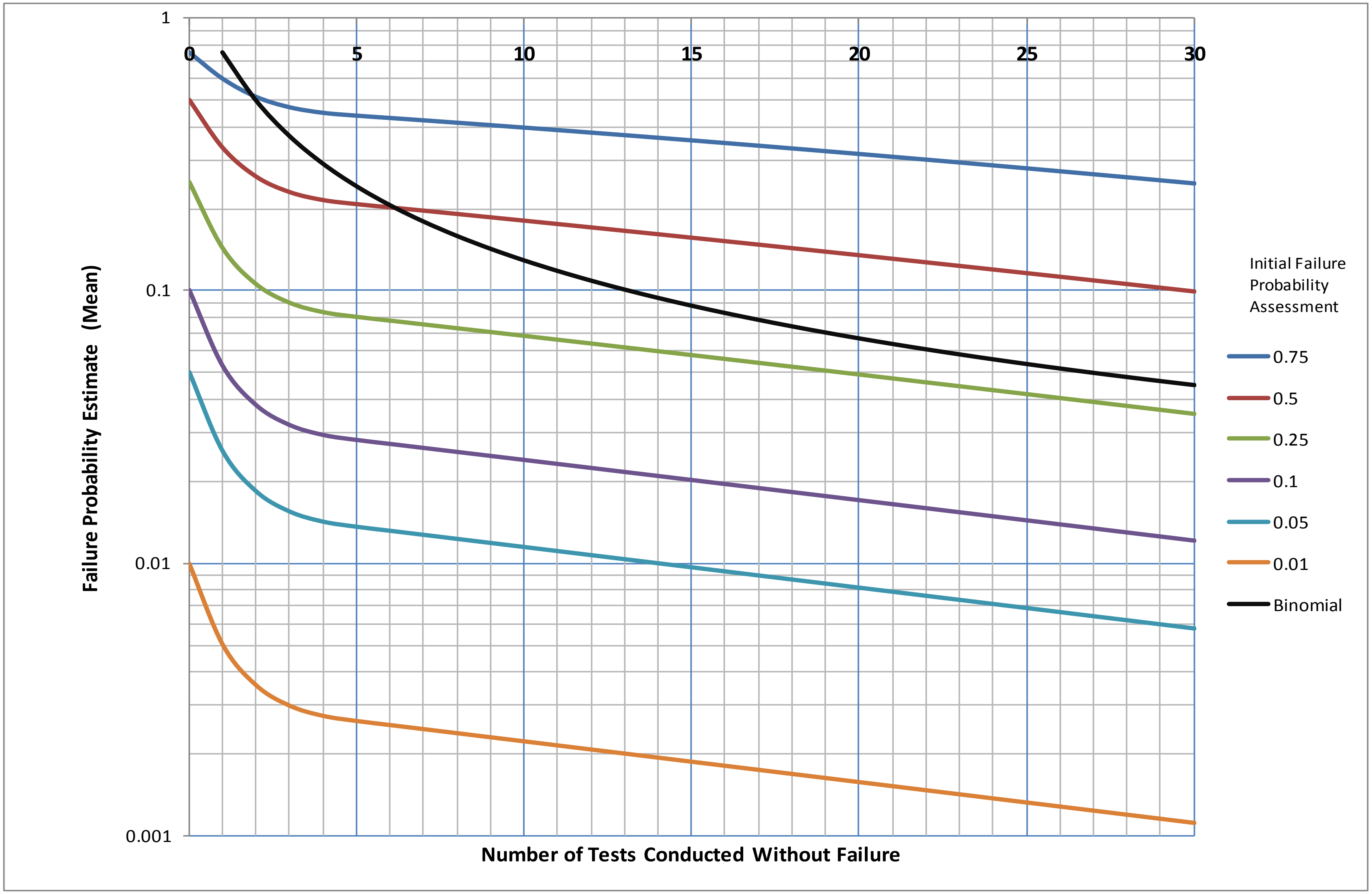
Applications to Determine the Number of Required Failure-Free Tests

- The next slide gives the number of required failure-free tests as a function of the initial assurance level
- The second slide overlays the curve for the binomial estimate which inaccurately treats the tests as throw-away tests
- The third slide quantifies the uncertainty and shows how it decreases with tests even if the initial information is uncertain
- These slides show the decision-making information provided using the Bayesian approach
- These results can be extended to cover the cases where failures or faults occur during the testing

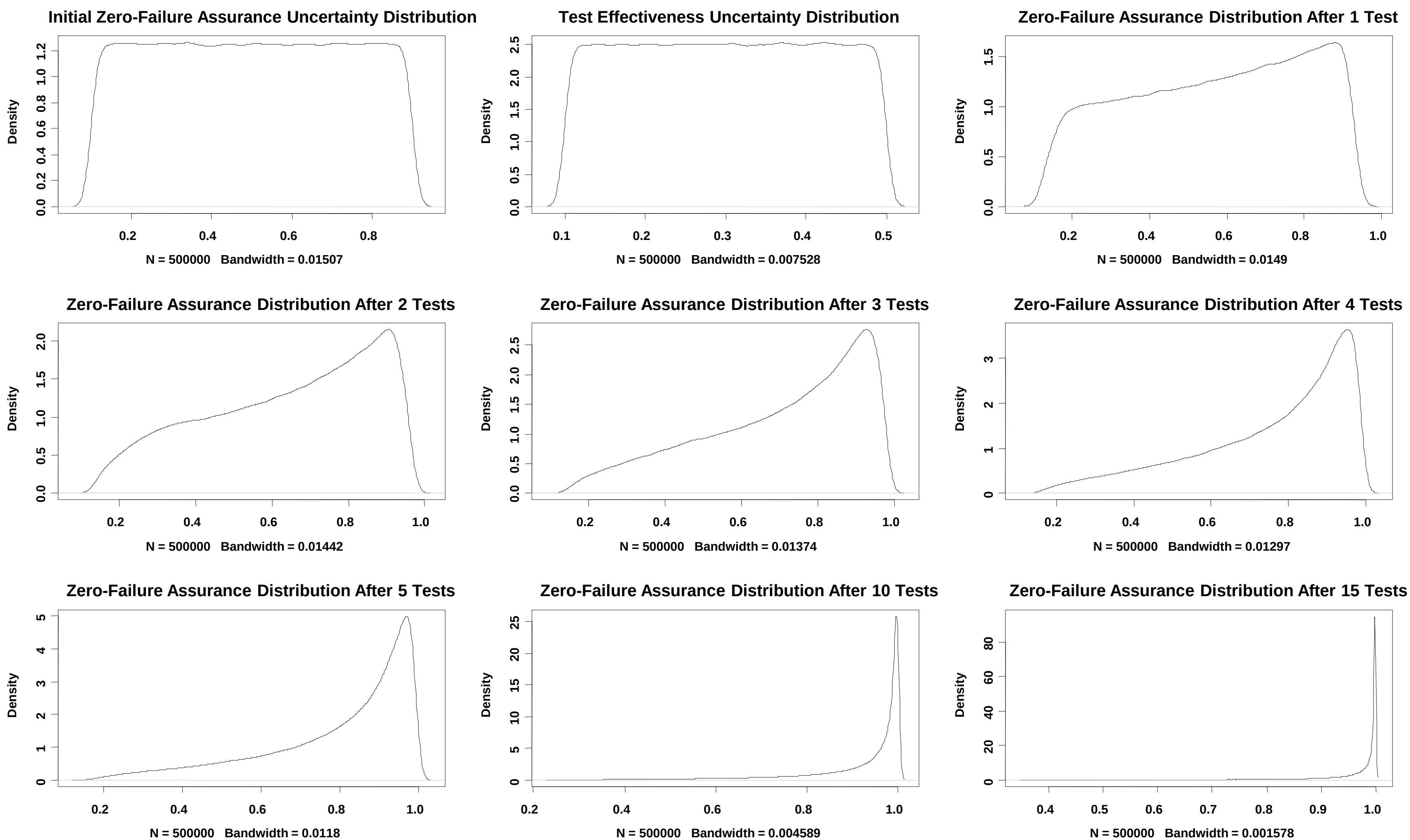
Failure Probability Estimates Versus Number of Failure Free Tests and Initial Failure Probability Estimate



Failure Probability Estimates Compared to the Binomial Sampling Estimate Which Models the System Tests as Throw-Away Sample Tests

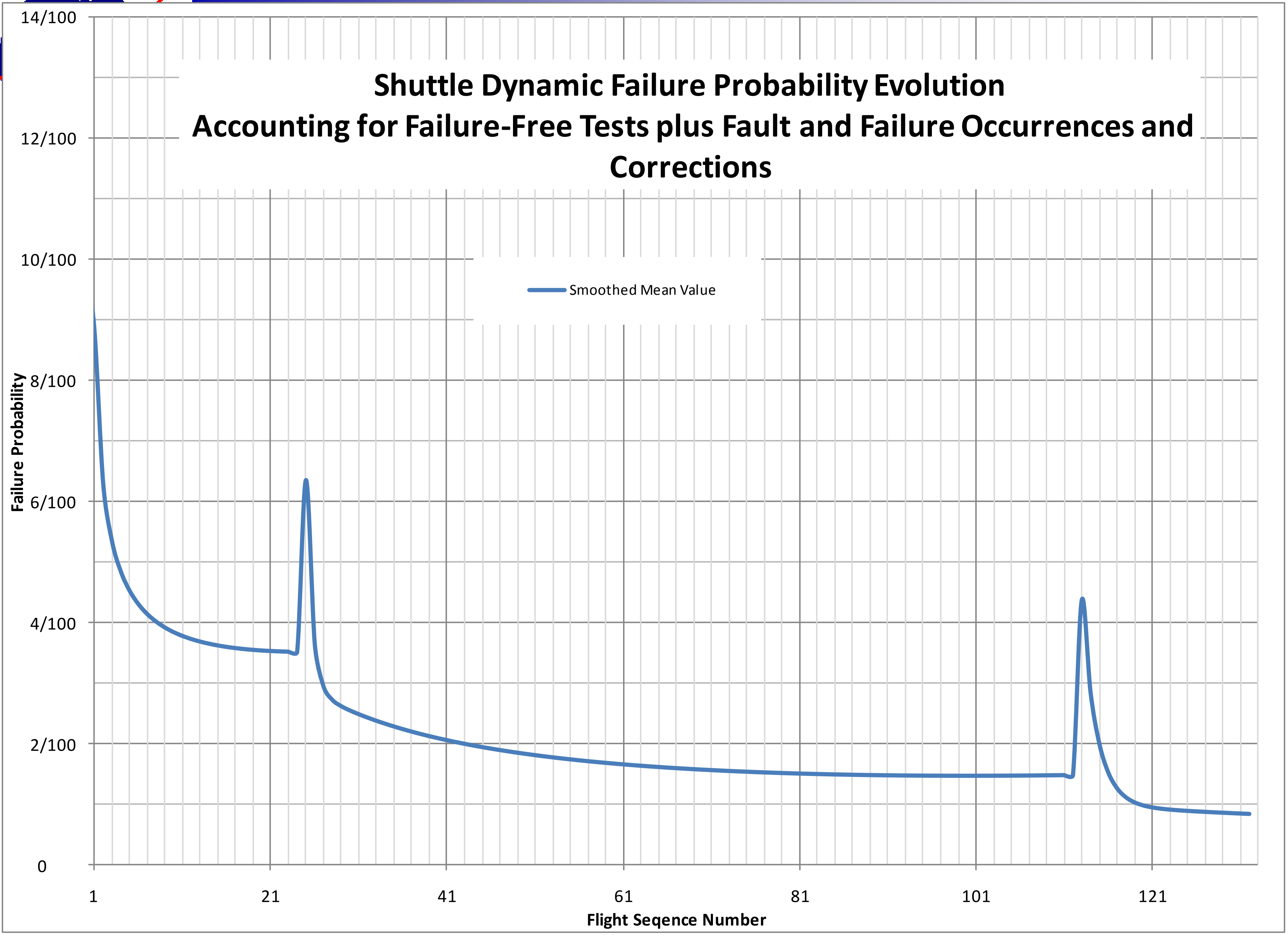


Uncertainty Distribution for the Reliability Estimate After Given Numbers of Tests



Application to Track Reliability Growth

- The next slide shows the application to track the reliability growth of the Space Shuttle
- The application updates the estimate for each next flight based on flight history information
- A Kalman filter is basically used on a transformed scale with Bayesian updating
- Both forward estimates and back estimates can be made
- Fault occurrences as well as failure occurrences are handled
- Software is developed to allow efficient application


Shuttle Dynamic Failure Probability Evolution
Accounting for Failure-Free Tests plus Fault and Failure Occurrences and Corrections
Smoothed Mean Value
Failure Probability
14/100
12/100
10/100
8/100
6/100
4/100
2/100
0
1
21
41
61
81
101
121
Flight Seqence Number

Summary

- **Key problems for a new system are the number of tests to conduct and the tracking of reliability**
- **The analysis needs to incorporate engineering information, reviews and oversights, and statistical data**
- **Bayesian analysis has these capabilities for dynamically updating estimates and quantifying uncertainties**
- **The application to number of tests needed shows the importance of incorporating the initial assurance level**
- **The application in tracking Shuttle shows the importance of dynamically tracking actual reliability growth**

Annotated References (1)

1. Gaver, D.P., and Jacobs, P.A., Testing or Fault Finding for Reliability Growth: A Missile Destructive-Test Example, Naval Research Logistics, Vol. 44 (1997), pp. 623-637 (Applies also to tests or flights with fault identification and correction.)
2. Sen, Ananda and Gouri, Bhattacharyya, A Reliability Growth Model Under Inherent and Assignable-Cause Failures, Balakrishnan, N. Recent Advances in Life-Testing and Reliability, CRC Press, 1995, pp. 295-311 (Statistical approach for separating faults and failures according to cause)
3. G. Glenn Shirley, A Defect Model of Reliability, 1995 International Reliability Symposium, Available at http://web.cecs.pdx.edu/~cgshirl/Glenns%20Publications/31%201995%20A%20Defect%20Model%20of%20Reliability%20IRPS95%20Tutorial%20Slides.pdf (Defect model applied to yield defects but methodology is detailed and generally applicable)

Annotated References (2)

4. Fenton, Norman, Neil, Martin, and Marquez, David, Using Bayesian Nets to Predict Software Defects and Reliability, Available at www.agenarisk.com/resource/white.../fentonMMR_Full_vi_0.pdf (Applicable to hardware defects also. Useful tool for aggregating information)
5. Walls, L.A. and Quigley, J.L., Building Prior Distributions to Support Bayesian Reliability Growth Modeling Using Expert Judgment, Reliability Engineering and System Safety, Vol. 74(2), (2001), pp.117-128 (Useful approach for assessing initial assurance level)
6. Li, Guo-Ying, Wu, Qi-Guang, and Zhao, Yong-Hur, Bayesian Analysis of Binomial Reliability Growth, Journal of Japan Statistics, Vol 32, No. 1 (2002), pp.1-14 (Applicable to both fault and failure counts)
7. ESAS, Final Report, NASA-TIM-2005-214062, November 2005, Chapter 8. Risk and Reliability, Appendix 8C. Reliability Growth (Reliability growth models used in NASA's ESAS study)
8. Kececioglu, Dimitri, Reliability Engineering Handbook, Volume 2, Simon and Schuster, 1991, Chapter 16. Reliability Growth (Basic reliability growth models)