Failure Domain Bounding with Applications to Dynamic Systems

Sean P. Kenny, Ph.D. and Daniel P. Giesy, Ph.D.
Dynamic Systems & Control Branch
NASA Langley Research Center

Luis G. Crespo, Ph.D.
Senior Research Scientist
National Institute of Aerospace

May 4, 2011

Favorite Quote from NSES 2011

"When systems fail, people notice."

Dr. J Michael Gilmore

Recent NASA History and Context

- **2003 – CAIB Report (Columbia accident):**
  - Engineering solutions should have included a quantifiable range of uncertainty and risk analysis.
- **2005 – RTFTG Final Report (Columbia accident):**
  - Further compounding the modeling challenge is the fact that the models are deterministic, yielding point estimates, without incorporating any measure of uncertainty in the result
- **2005 – NASA CEV RFP**
  - Design and execute a meaningful risk mitigation program that culminates in a risk reduction flight effort and PDR by the end of calendar year 2008
- **2007 – NASA-STD-7009 for Models & Simulations:**
  - The risk assumed by the decision maker is often misestimated due to inadequate assessment of uncertainties
  - Reports to decision makers of M&S results shall include an estimate of their uncertainty and a description of any processes used to obtain this estimate

Uncertainty Analysis and Robust Design

Increase confidence and consistency in aerospace vehicle safety predictions by developing improved methods for quantifying and managing uncertainty

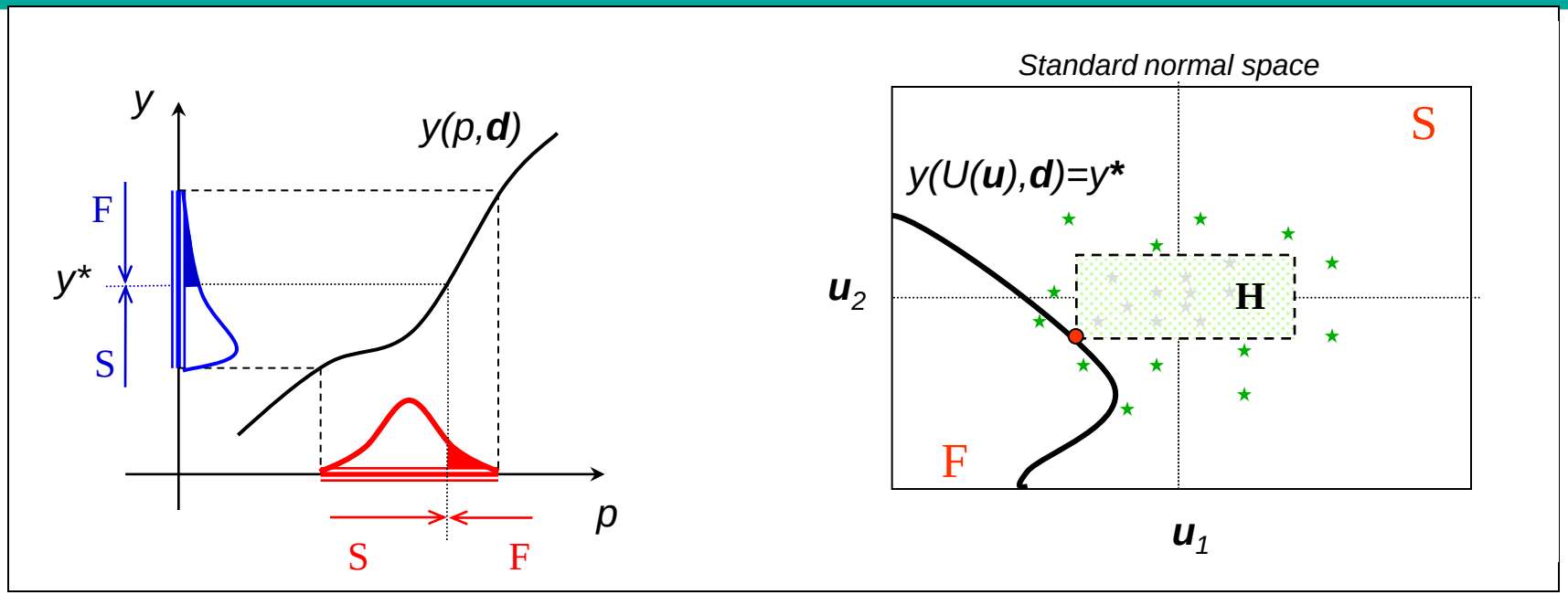
Quantifying

- Uncertainty Modeling
  - *Model uncertainty based on experimental data, simulations and/or expert opinion*
- Uncertainty Propagation
  - *Given uncertainty models of a system's inputs, how to propagate them through system models, to efficiently evaluate the corresponding system's outputs?*

Managing

- Robust Design
  - *Generate designs that robustly accommodate uncertainty*
- Uncertainty Decomposition
  - *Identify uncertainties that contribute the most to performance degradation*
  - *Determine the parameters that should (not) be modeled as uncertain*

Failure Domain Bounding via Homothetic Deformations

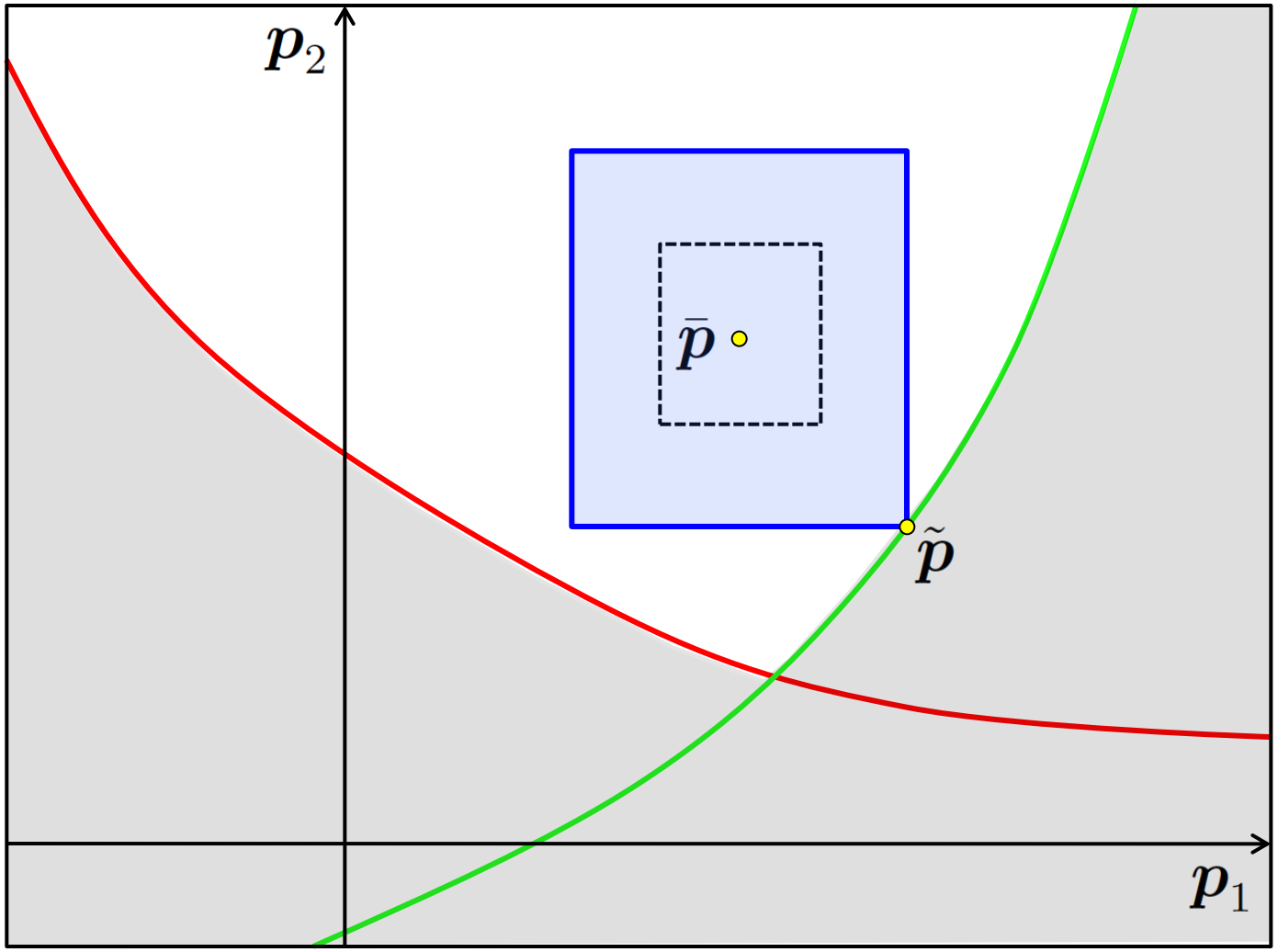


- Monte Carlo sampling
- Homothetic deformations
  - Optimization-based
  - Computationally cheap
  - Analytical expressions for P[H]

Applications

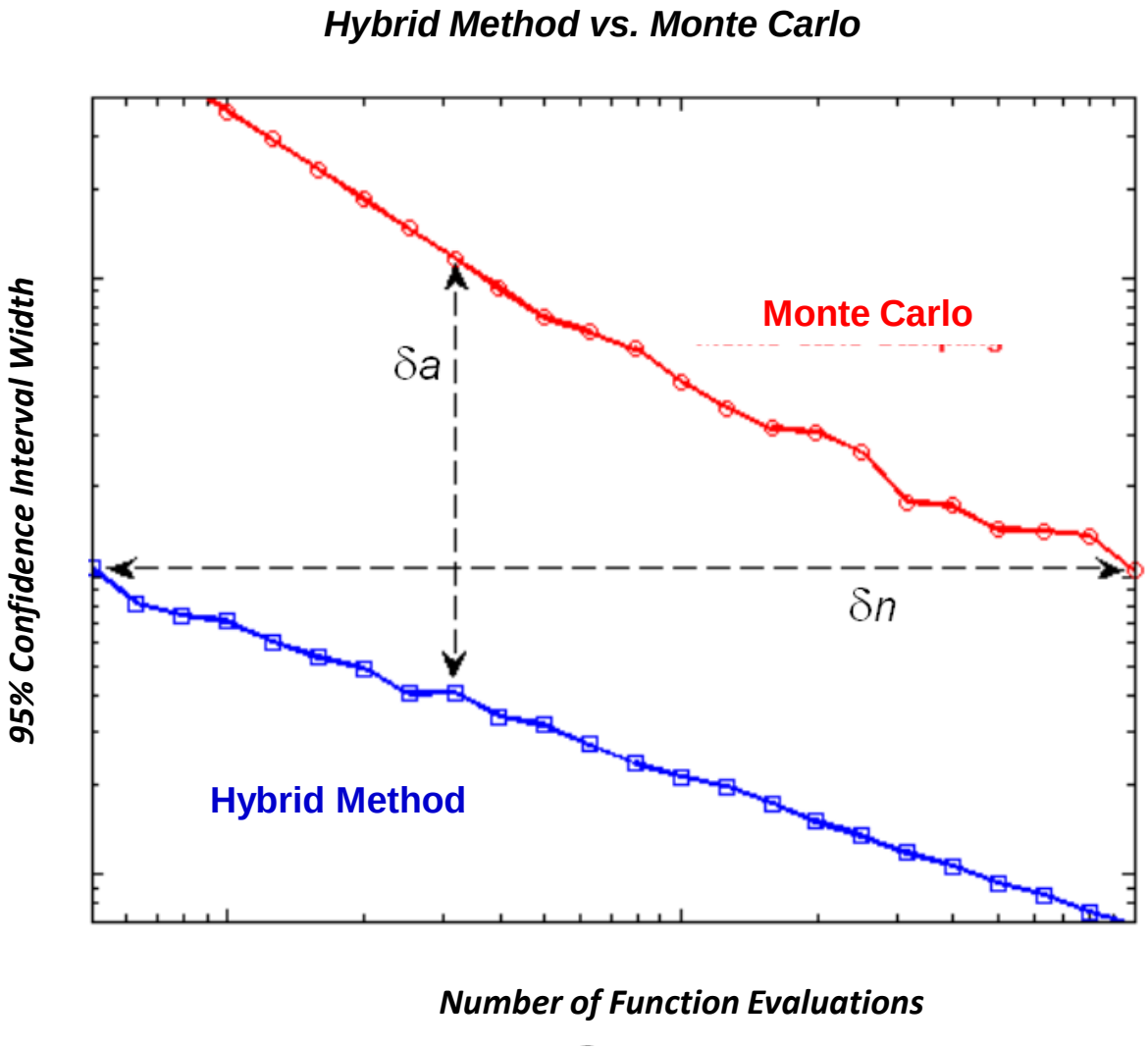
1. Robustness metric → PSM
2. Upper bounds to P[F]
3. Hybrid Method for P[F]

Analysis: Homothetic Deformations



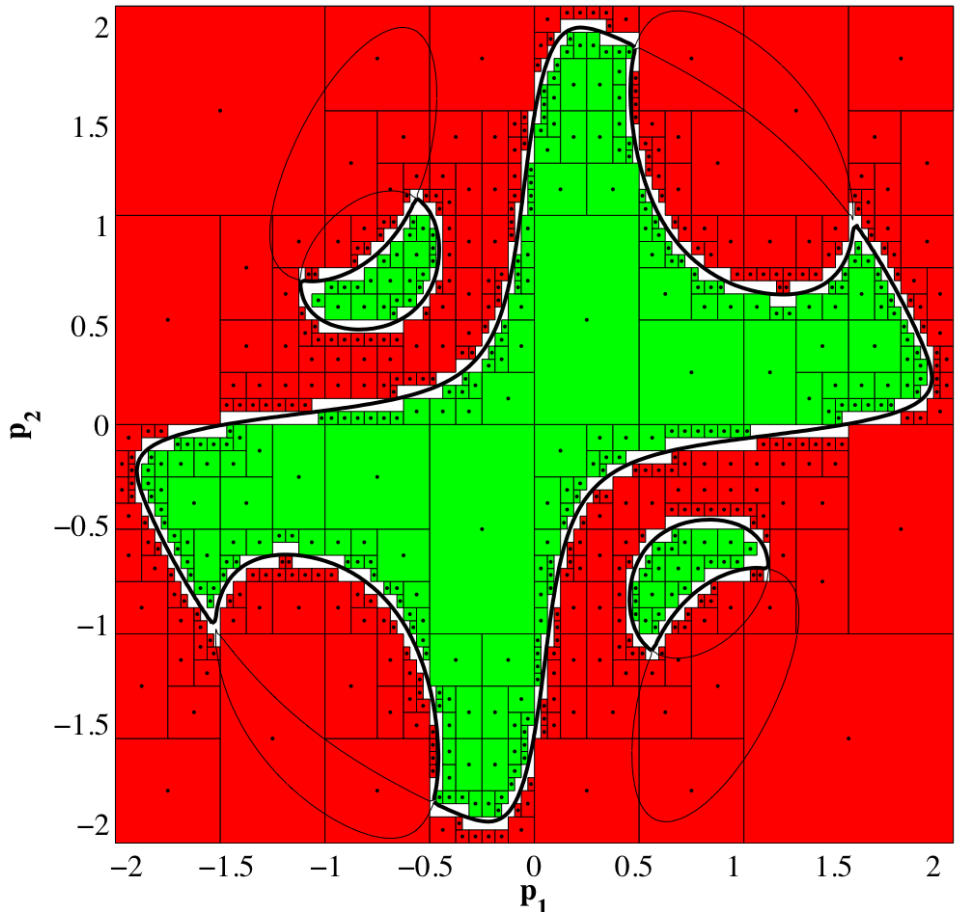
- ***Outcomes: robustness metric, worst-case uncertainty, separation, probability bounds***

Efficiency Relative to Monte Carlo

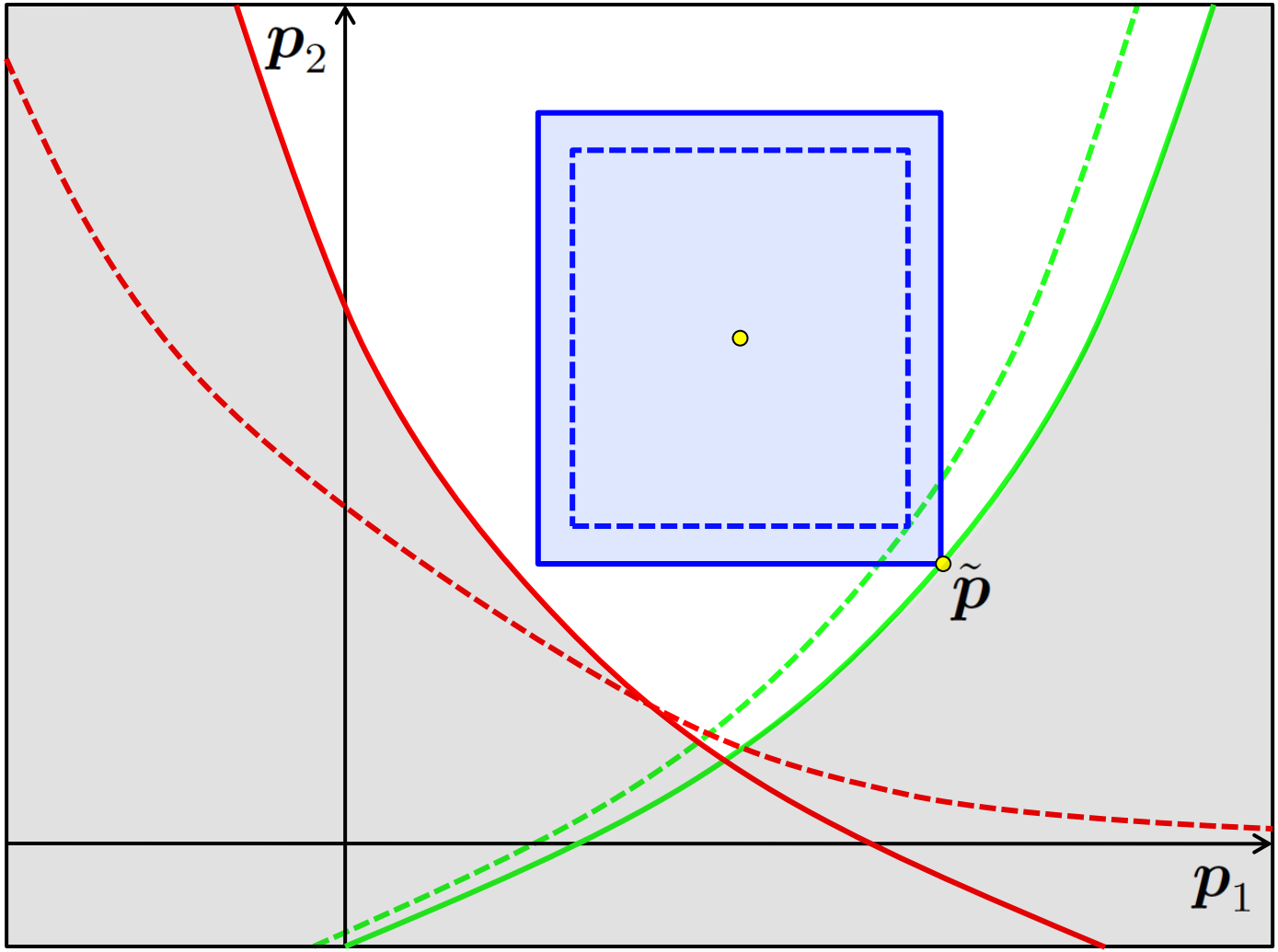


Failure Domain Approximation

- Builds upon the research in homothetic deformations.
- Yields high fidelity characterizations of complex nonlinear failure domains.
- Utilizes theory of Bernstein polynomials.
- Desensitizes the analysis from assumptions used to model the uncertainty.



Design: Homothetic Deformations



- ***Conflicting objectives, optimally robust designs***

The Tool Suite: UQTools

UQTools is a collection of Matlab functions designed to quantify the impact of uncertainty on generic, continuous, parameterized models.

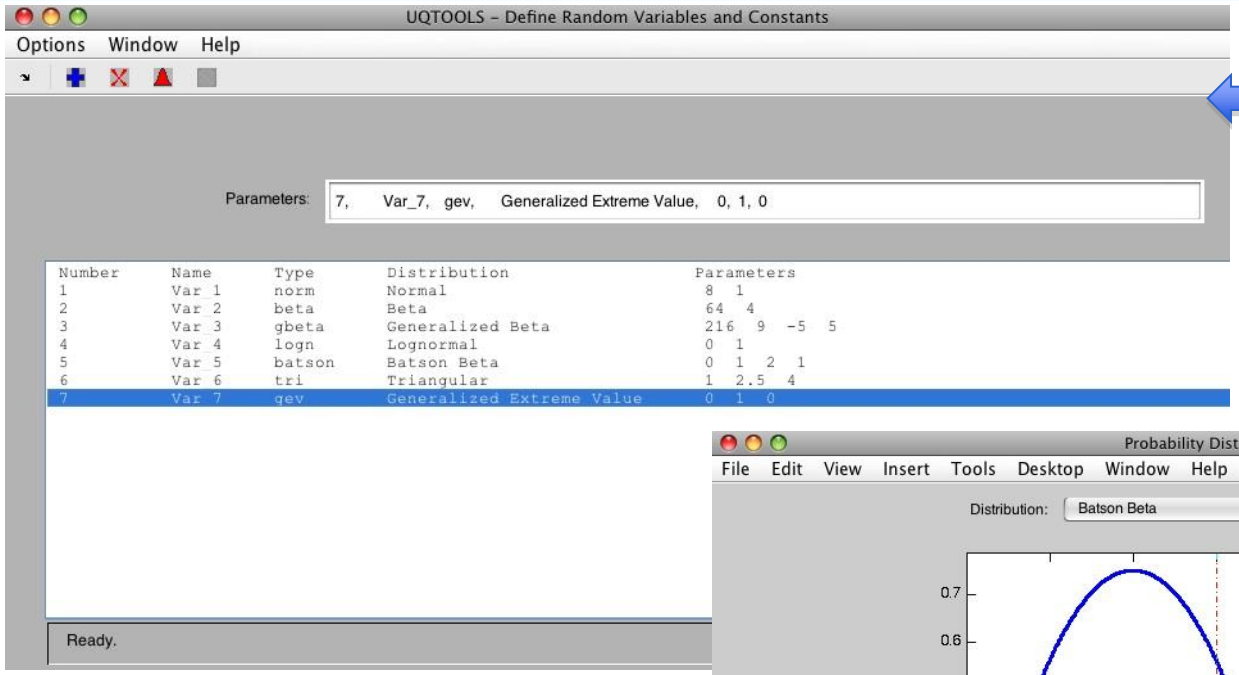
- Uncertainty Models:
  - Probability density functions
  - Non-probabilistic sets (hyper-rectangular and hyper-spherical)
- System Models:
  - Matlab-callable parameterized input/output maps.

UQTools Capabilities

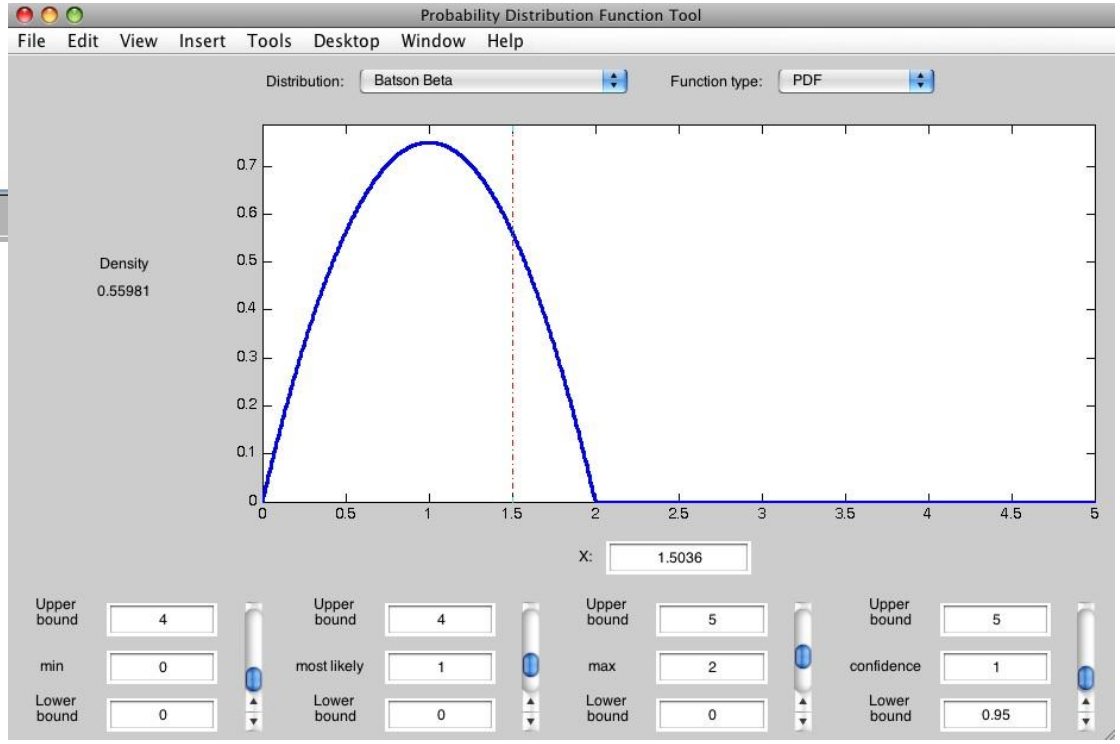
Integration of a collection of tools (each piece designed to attack a specific issue in UQ). The integration represents a unique capability in the field.

- Efficient methods for failure set bounding
  - Optimization-based approach for computing upper bounds on failure domains
- Hybrid methods for efficient estimation of failure probabilities
  - Combining failure set bounding theory with efficient conditional sampling
- First-Order Reliability Method
  - Efficient failure probability approximation for low probability 'tail' events
- Efficient deterministic sampling
  - Substantial improvement over conventional Monte Carlo
- Efficient moment propagation methods
  - Useful for propagating trends, e.g., mean & variance of system response
- Probabilistic sensitivity analysis
  - Analyze and rank the relative importance of system parameters
- Response surface tools
  - Radial basis functions and generalized polynomials (with 1st and 2nd derivatives)

Graphical User Interface



Master Parameter List Interface



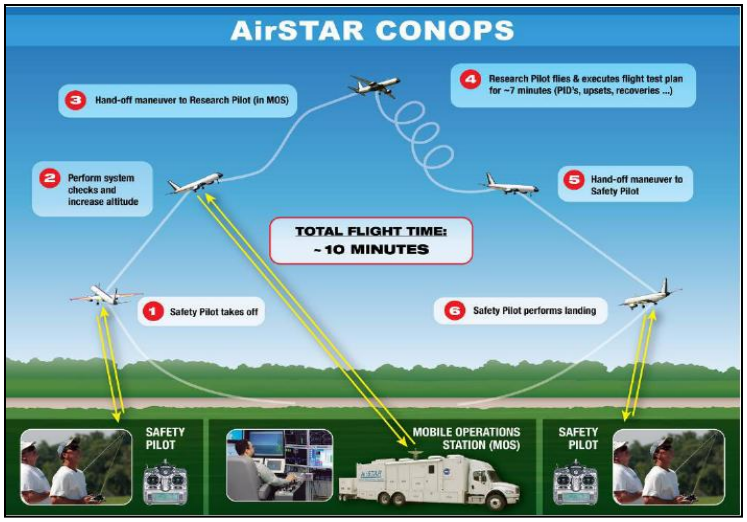
Density Function Utility

- ***Currently 20 different distributions supported.***
- ***General intervals and deterministic parameters also supported.***

Example Applications

Example: GTM Control Analysis & Design

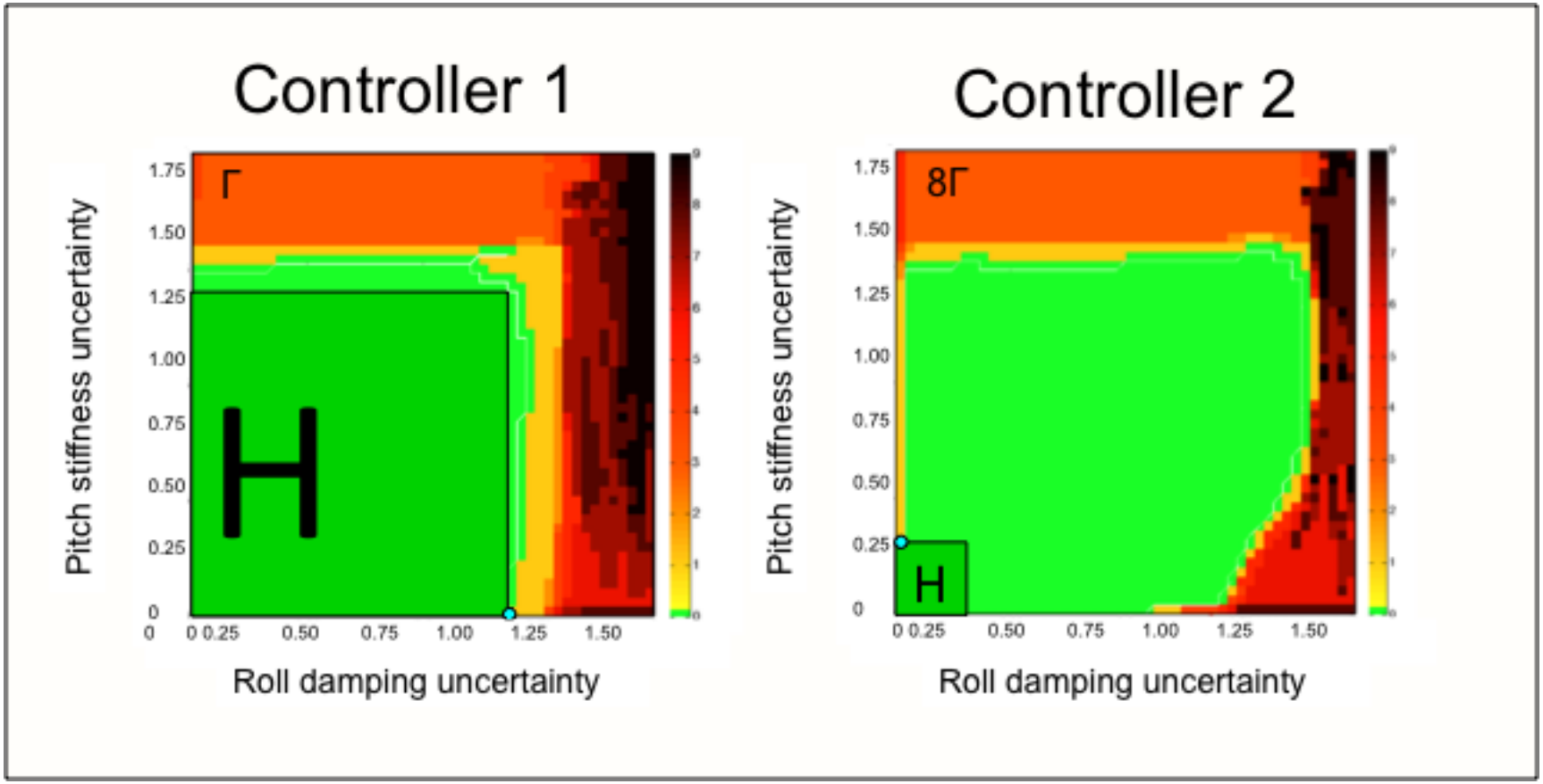
- Subscale, physical model
- High-fidelity Simulink model: non-linear aero, avionics, engine and sensor dynamics, atmospheric model, telemetry effects, time delay, filters, etc. (278 states)



- Control structure: LQR-PI and Model Reference Adaptive
- Uncertain parameters: aerodynamic coefficients
- Requirements: structural integrity, reliable flight envelope, command following, high frequency/residual oscillation

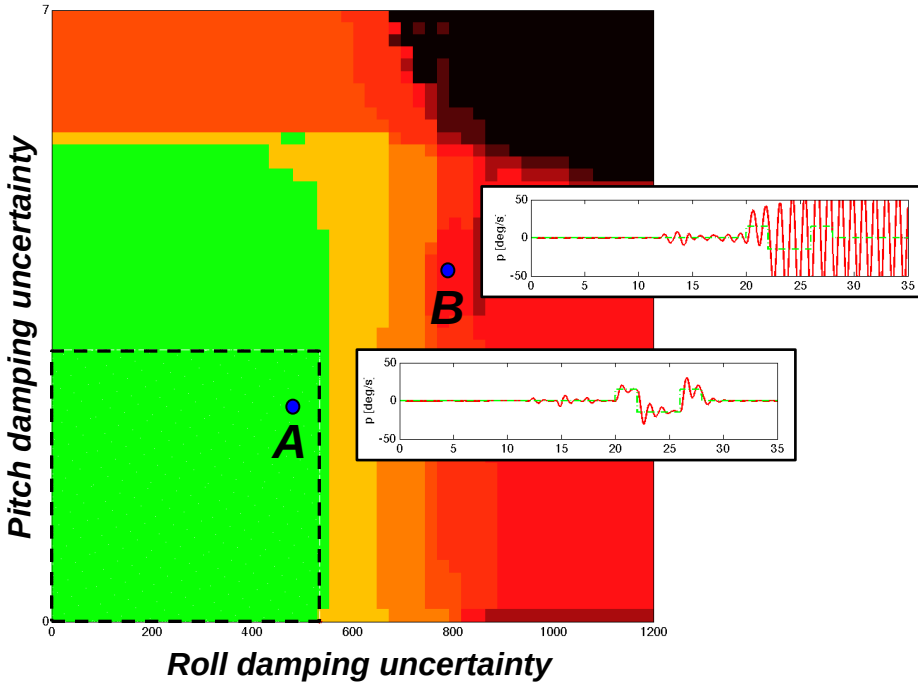
Homothetic Deformations: Analysis

- Two different controllers analyzed for robustness to aero uncertainties.
- Despite having a larger safe operating region, Controller 2 (high gain controller) has a nearly undetectable failure mode close to nominal point.
- Conventional Monte Carlo is not well-suited to capture this type of failure.

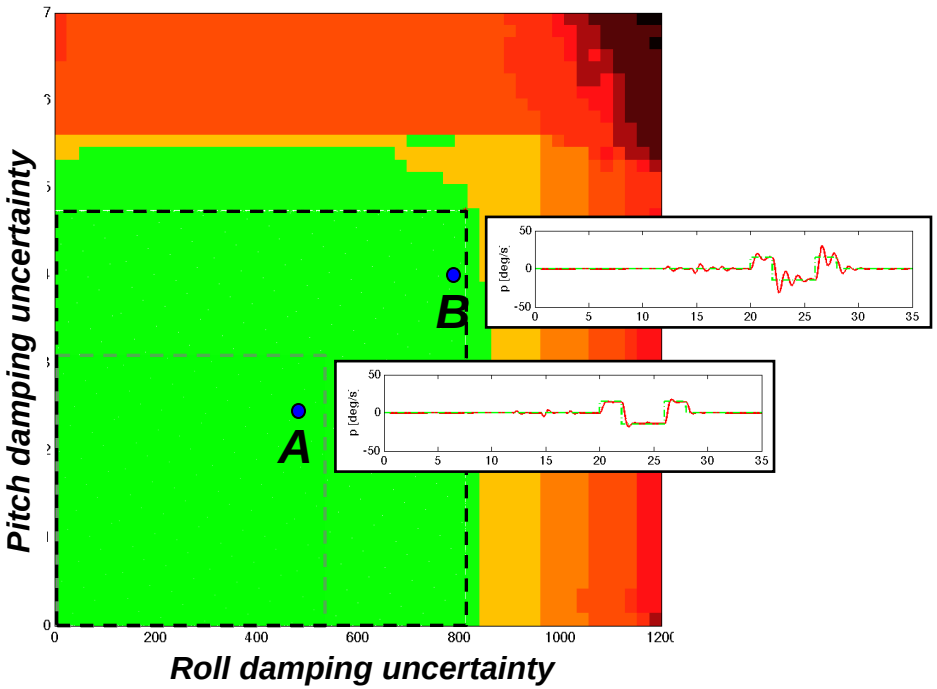


Homothetic Deformations: Design

Before Redesign

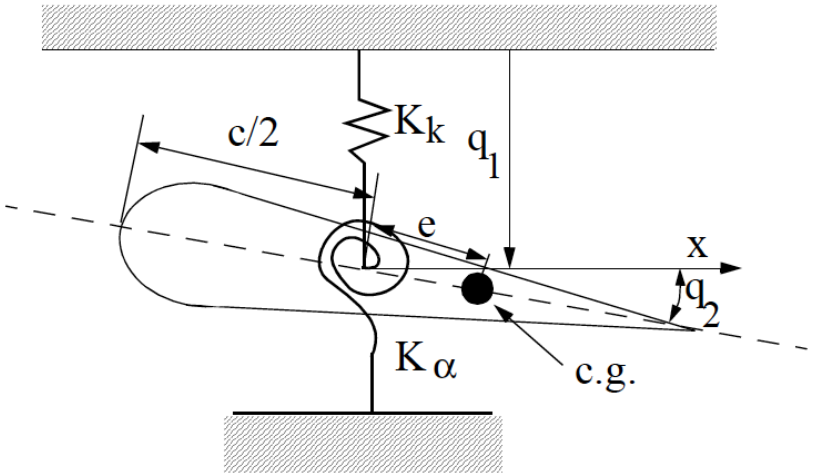


After Redesign



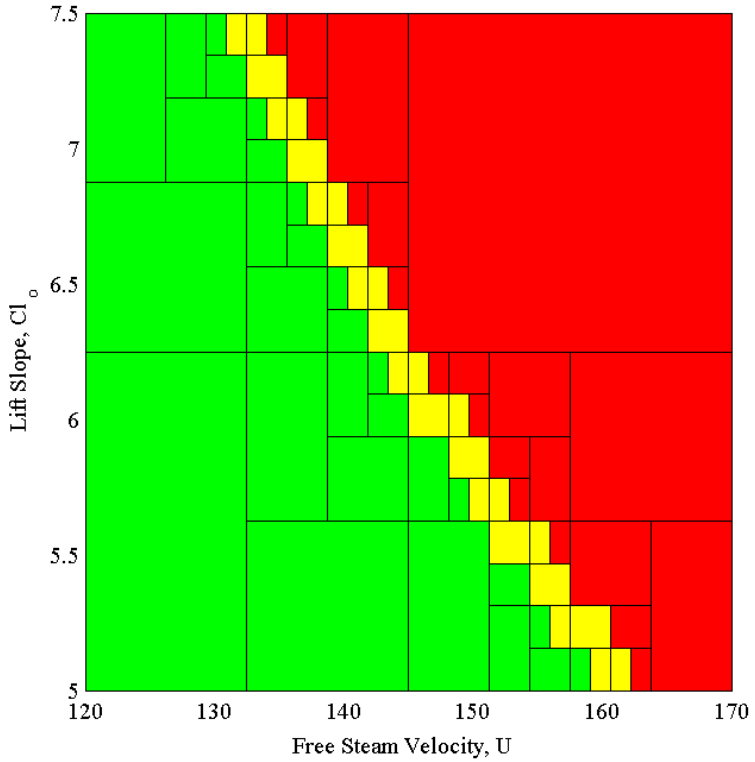
65 % improvement *in* roll *d*amping uncertainty

Simple Aeroelastic Model

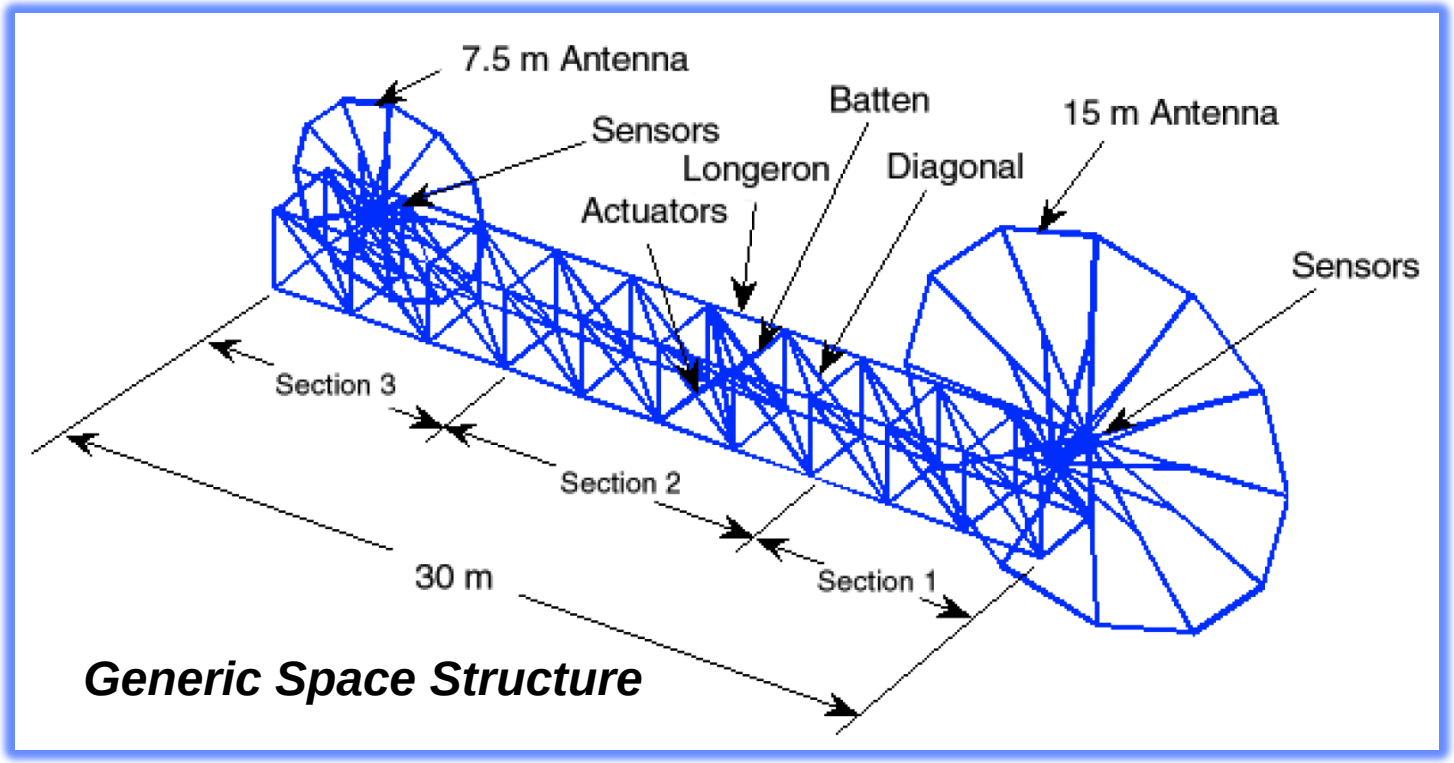


- Linear Aeroelasticity
- 4 uncertain parameters
- Bernstein polynomials
- 5 Routh-Hurwitz constraints

$$A(U, K_k, K_\alpha, Cl_\alpha) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-10K_k}{9}\left(\frac{-245Cl_\alpha U^2}{288}+\frac{10K_\alpha}{9}\right) & \frac{-245Cl_\alpha U}{288} & 0 & \\ \frac{10K_k}{9}\left(\frac{343Cl_\alpha U^2}{144}-\frac{100K_\alpha}{9}\right) & \frac{343Cl_\alpha U}{144} & 0 & \end{bmatrix}$$

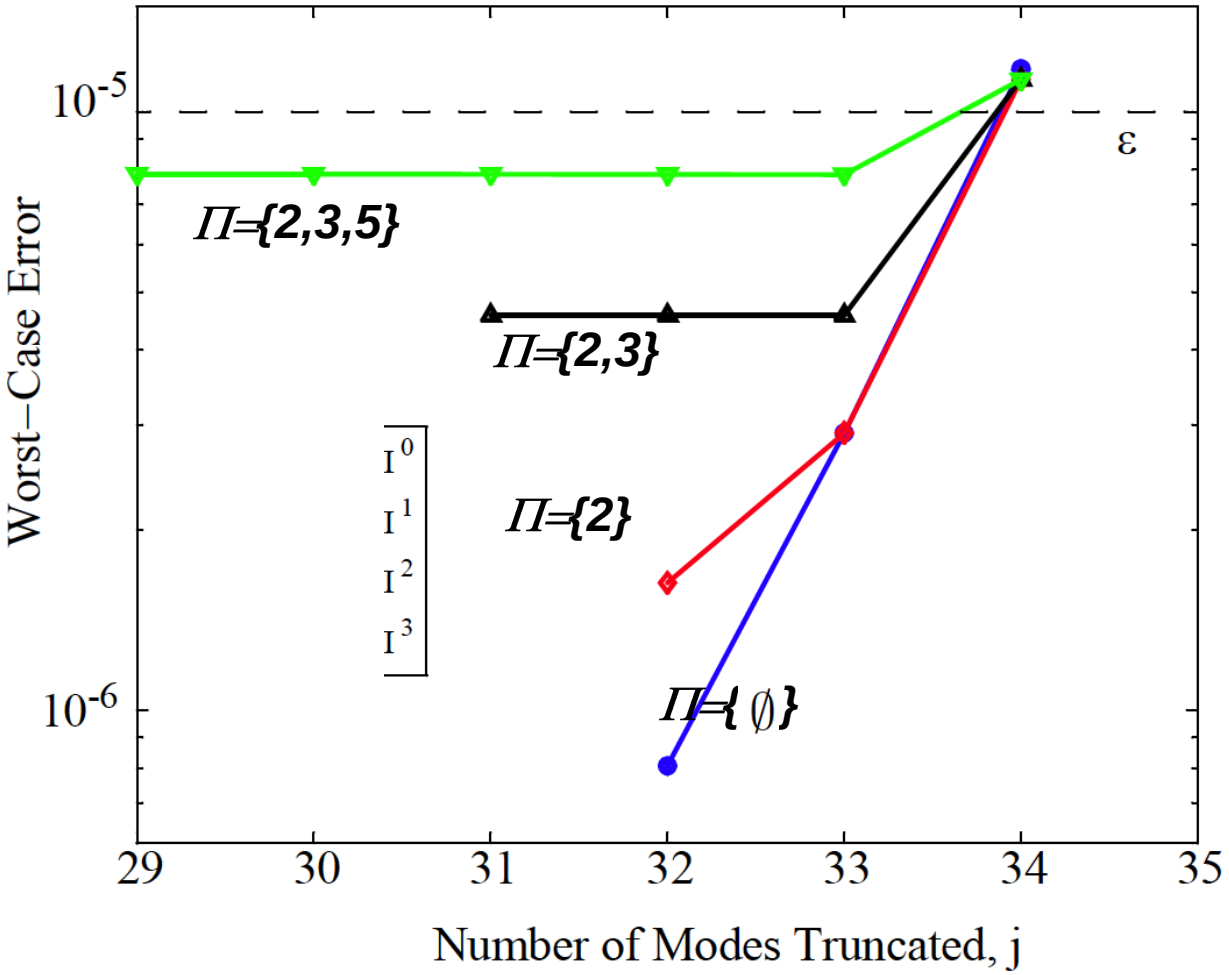


Example: Flexible Antenna



- Uncertain parameters: outer radius of precision tube members (14 groups, with Δp = +/- 0.5 %)
- Maximum allowable error, $\varepsilon=10^{-5}$, (-100 dB)
- MIMO system (78 states, 3 inputs, 6 outputs)

Example: Flexible Antenna



Efforts to Improve State of Practice

- In June 2009, LaRC formed an Uncertainty Based Methods Community Forum (currently 60 members – one AMES CS + one private sector researcher)

  The objective of the UBM Community Forum is to facilitate the cross fertilization of methods, tools, and ideas related to statistical and probabilistic analysis and design for a broad spectrum of engineering applications

- Foster informal and open discussion of problems in uncertainty based methods
- Discuss success and/or failure
- Share current applications
- Provide a forum for new methods to be vetted

Conclusions

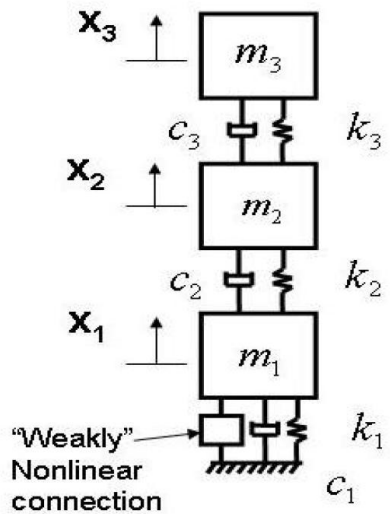
- ***High-fidelity characterization of the failure domain***
- ***Identification of worst-case uncertainty combinations***
- ***Exact failure probability bounds***
- ***Substantially desensitizes the uncertainty analysis from the uncertainty model assumed***

Validation Challenge Workshop

Sandia National Laboratory Workshop

(13 international teams of experts chosen to participate)

Challenge: Adequate statistical characterization of uncertainty using limited experimental data.

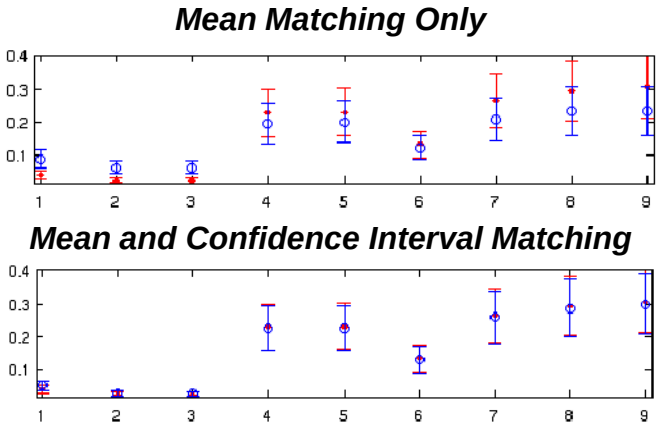


Uncertain/Nonlinear Subsystem

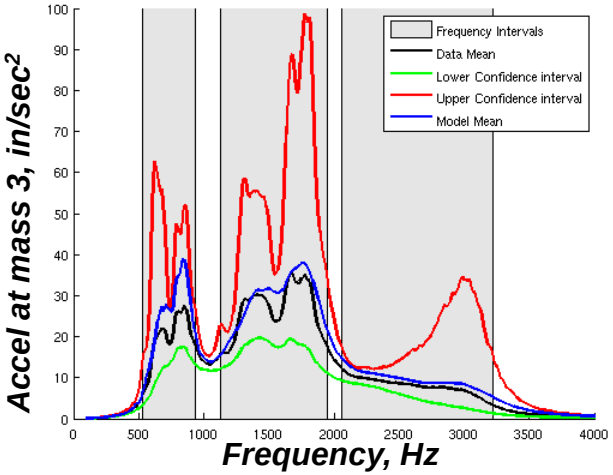
40 data points (2 sets of 20)

LaRC's Solution

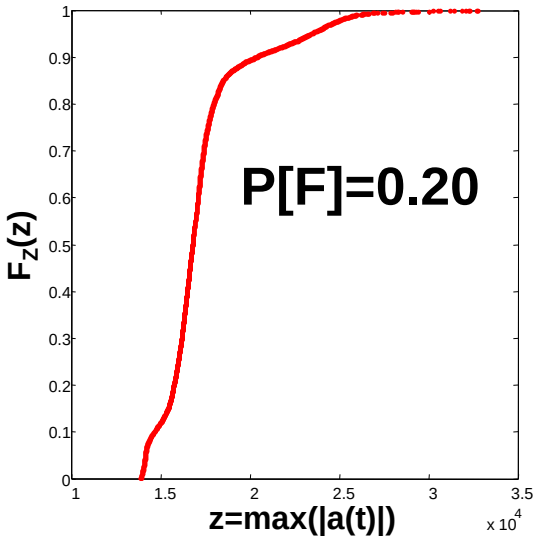
Statistical matching of parametric uncertainties



Statistical response of uncertainty model versus data



Results on Target System



Regulatory Requirement

$P[z>18000\ in/sec^2] < 0.01$